# Vacuum technology
# Kinetic gas theory

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## The goal of kinetic gas theory

- Description of gases/bulk by solving equations of motion of individual particles
- Vast number of particles ($N \sim 10^{23}$) : 3N differential equations, 6N initial values $\Rightarrow$ statistical approach (physical quantities, p, V, T, n… matter rather than ind. trajectories)
- Molecules are described in state space $(q, \dot{q}, t)$ or phase space $(q, p, t)$ by means of probability density functions $f(q, \dot{q}, t)$ or $f(q, p, t)$



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## What can we gain?

- Equilibrium is more easily described in the general theory (statistical mechanics)
- The number of collisions is limited in vacuum $\Rightarrow$ individual collisions become important
- Some tricks can help in special calculations
- Transport processes



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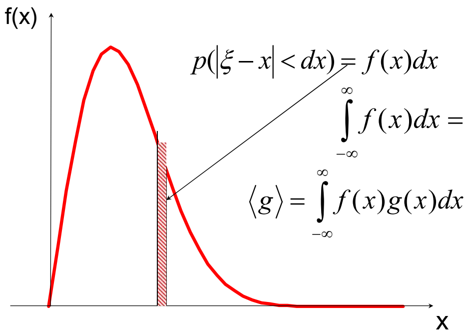


$$p(|\xi - x| < dx) = f(x)dx$$

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\langle g \rangle = \int_{-\infty}^{\infty} f(x)g(x)dx$$



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## Properties of the probability density function

$$f(x) \xrightarrow{\sim ? \sim} f'(y)$$

$$f(x)dx = f'(x(y))\frac{dx}{dy}dy$$

$$f(\underline{x})d\underline{x} = f'(\underline{x}(\underline{y}))\left|\underline{\underline{J}}\right|d\underline{y} \qquad \left|\underline{\underline{J}}\right| = \begin{vmatrix} \partial x_1/\partial y_1 & \cdots & \partial x_n/\partial y_1 \\ \vdots & \ddots & \vdots \\ \partial x_1/\partial y_n & \cdots & \partial x_n/\partial y_n \end{vmatrix}$$



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## Starting point

- Particle–particle potential decays fast$\rightarrow$ hard sphere model, only collisions
- In equilibrium, in the lack of external potential the distribution of the particles is homogeneous, isotropic



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## Determination of the density function I.

- $F(x,y,z,v_x,v_y,v_z)=F(v_x,v_y,v_z)=f_x(v_x)f_y(v_y)f_z(v_z)$
- $\ln F(v_x,v_y,v_z)=\ln f_x(v_x)+\ln f_y(v_y)+\ln f_z(v_z)$ (independent events)

$$\frac{\partial \ln F}{\partial v_x}=\frac{d\ln f_x(v_x)}{dv_x}$$

$$v=\sqrt{v_x^2+v_y^2+v_z^2}$$

$$\frac{\partial \ln F}{\partial v}\cdot\frac{\partial v}{\partial v_x}=\frac{d\ln f_x(v_x)}{dv_x}$$

$$\frac{\partial \ln F}{\partial v}\cdot\frac{v_x}{v}=\frac{d\ln f_x(v_x)}{dv_x}$$

$$v^2=v_x^2+v_y^2+v_z^2$$

$$\frac{\partial v^2}{\partial v}\cdot\frac{\partial v}{\partial v_x}=\frac{dv_x^2}{dv_x}=2v_x$$

$$2v\cdot\frac{\partial v}{\partial v_x}=2v_x$$

$$\frac{\partial v}{\partial v_x}=\frac{v_x}{v}$$



## Determination of the density function II.

$$\frac{\partial \ln F}{\partial v}\cdot\frac{1}{v}=\frac{d\ln f_x(v_x)}{dv_x}\cdot\frac{1}{v_x}=\frac{d\ln f_y(v_y)}{dv_y}\cdot\frac{1}{v_y}=\cdots$$

$$\frac{d\ln f_x(v_x)}{dv_x}\cdot\frac{1}{v_x}=-\beta$$

$$d\ln f_x(v_x)=-\beta\cdot v_x dv_x$$

$$\ln f_x(v_x)=-\frac{\beta}{2}\cdot v_x^2+\ln a$$

$$f_x(v_x)=\alpha\cdot e^{-\frac{\beta}{2}v_x^2}$$



## Determination of the density function III. (α)

$$f_x(v_x)=\alpha e^{-\frac{\beta}{2}\cdot v_x^2}$$

$$1=\int_{-\infty}^{\infty} f_x(v_x)dx=\alpha\int_{-\infty}^{\infty} e^{-\frac{\beta}{2}v_x^2}dx=\alpha\sqrt{\frac{2\pi}{\beta}}$$

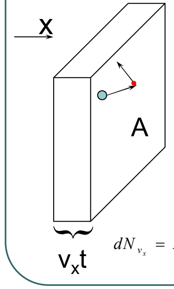
$$\int_{-\infty}^{\infty} e^{-ax^2}dx=\sqrt{\frac{\pi}{a}}$$

$$\alpha=\sqrt{\frac{\beta}{2\pi}}$$

$$f_x(v_x)=\sqrt{\frac{\beta}{2\pi}}\cdot e^{-\frac{\beta}{2}v_x^2}$$



## Determination of the density function IV. (β)




$$\Delta(momentum)=2mv_x$$ 1 molecule

$$F=\frac{d(momentum)}{dt}$$

pressure

$$p=\frac{F}{A}=z^{wall}\cdot\Delta(momentum)$$

$$p_{v_x}=\frac{F_{v_x}}{A}=2\cdot z_{v_x}^{fal}\cdot mv_x$$

$$\frac{Wall-collisions}{A\cdot t}$$

$$\frac{dN_{v_x}}{V}\cdot A\cdot v_x t$$ Molecules reach the wall

$$dN_{v_x}=N\cdot f_x(v_x)dv_x \Longrightarrow z_{v_x}^{fal}=\frac{N}{V}\cdot f_x(v_x)\cdot v_x dv_x$$



## Determination of the density function IV. (β)

$$p_{v_x}=2\cdot z_{v_x}^{fal}\cdot m\cdot v_x=2\left(\frac{N}{V}\cdot f_x(v_x)dv_x\cdot v_x\right)mv_x=\frac{2N\cdot m}{V}v_x^2 f_x(v_x)dv_x$$

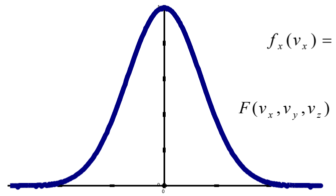
$$pV=2N\cdot m\int_0^{\infty} v_x^2 f_x(v_x)dv_x=Nm\int_{-\infty}^{\infty} v_x^2 f_x(v_x)dv_x$$

$$pV=\frac{N}{N_A}RT=NkT \Longrightarrow \int_{-\infty}^{\infty} v_x^2 f_x(v_x)dv_x=\frac{kT}{m}$$

$$\frac{kT}{m}=\sqrt{\frac{\beta}{2\pi}}\int_{-\infty}^{\infty} v_x^2 e^{-\frac{\beta}{2}v_x^2}dv_x=\sqrt{\frac{\beta}{2\pi}}\cdot\frac{\sqrt{\pi}}{2\left(\frac{\beta}{2}\right)^{\frac{3}{2}}}=\frac{1}{\beta} \Longrightarrow \beta=\frac{m}{kT}$$



## The density function(s)



$$f_x(v_x)=\sqrt{\frac{m}{2\pi\cdot kT}}e^{-\frac{mv_x^2}{2kT}}$$

$$F(v_x,v_y,v_z)=\left(\frac{m}{2\pi\cdot kT}\right)^{3/2}e^{-\frac{m(v_x^2+v_y^2+v_z^2)}{2kT}}$$

Even functions… what's happened?

We are rather inteerested in <|v|>!

## Some more functions...

For the distribution of |v| transform it to polar coordinates

$$\hat{f}(v,\vartheta,\varphi) = \left|\underline{\underline{J}}\right| F(v) = \begin{vmatrix} \partial vx/\partial v & \partial vy/\partial v & \partial vz/\partial v \\ \partial vx/\partial\vartheta & \partial vy/\partial\vartheta & \partial vz/\partial\vartheta \\ \partial vx/\partial\varphi & \partial vy/\partial\varphi & \partial vz/\partial\varphi \end{vmatrix} F(v) =$$

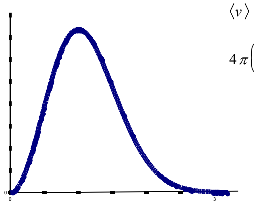
$$= \left(v^2 \sin\vartheta\right)\left(\frac{m}{2\pi\cdot kT}\right)^{3/2} e^{-\frac{mv^2}{2kT}}$$

$$f(v) = \int_0^{\pi}\int_0^{2\pi} \hat{f}(v,\vartheta,\varphi)\,d\varphi\,d\vartheta = \int_0^{\pi}\sin\vartheta\,d\vartheta\int_0^{2\pi}d\varphi\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}} =$$

$$= 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}}$$ Maxwell- velocity distribution



## Getting better...



$$\langle v\rangle = 4\pi\left(\frac{m}{2\pi kT}\right)^{2/3}\int_0^{\infty} v\cdot v^2 e^{-\frac{mv^2}{2kT}}dv =$$

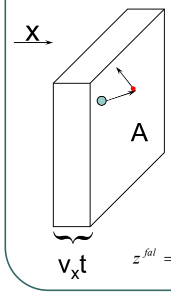
$$4\pi\left(\frac{m}{2\pi kT}\right)^{2/3}\frac{1}{2\left(\frac{m}{2kT}\right)^2} = \sqrt{\frac{8kT}{\pi m}}$$

$$v_{rms} = \sqrt{\langle v^2\rangle} = \sqrt{\frac{3kT}{m}}$$

$$v_m = \sqrt{\frac{2kT}{m}}$$



## Number of wall-collisions




$$\frac{dN_{v_x}}{V}\cdot A\cdot v_x t$$ Molecules reach the wall

$$dN_{v_x} = N\cdot f_x(v_x)dv_x \Longrightarrow z_{v_x}^{fal} = \frac{N}{V}\cdot f_x(v_x)\cdot v_x dv_x$$

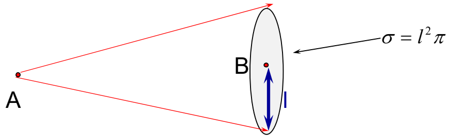
$$\int_0^{\infty} f_x v_x dv_x = \sqrt{\frac{m}{2\pi\cdot kT}}\int_0^{\infty} v_x e^{-mv_x^2/2kT}dv_x =$$

$$= \sqrt{\frac{m}{2\pi\cdot kT}}\frac{kT}{m} = \sqrt{\frac{kT}{2\pi\cdot m}} = \frac{1}{4}\langle v\rangle$$

$$z^{fal} = \frac{N}{4V}\cdot\langle v\rangle = \frac{N}{V}\sqrt{\frac{kT}{2\pi\cdot m}} = \frac{p}{kT}\sqrt{\frac{kT}{2\pi\cdot m}} = \frac{p}{\sqrt{2\pi\cdot mkT}}$$



## Molecular collisions




- Any molecule collides with a maximum of 1 other one
- Relative velocity metters, it has a characteristic distribution, calculated from Maxwell-distribution
- Number of collisions (/s): $\int_0^{\infty}\sigma v_r \frac{N}{V} f(v_r)dv_r$



## The distribution of the relative velocity

- Look at two molecules. The probability that the velocity of the first one is between $\underline{v}_1$ and $\underline{v}_1+\underline{dv}_1$, and the others are between $\underline{v}_2$ and $\underline{v}_2+\underline{dv}_2$ (independent events):

$$f_{12}(\vec{v}_1,\vec{v}_2)d\vec{v}_1 d\vec{v}_2 = \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{mv_1^2}{2kT}}dv_{1x}dv_{1y}dv_{1z}\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{mv_2^2}{2kT}}dv_{2x}dv_{2y}dv_{2z}$$

- The location vectors of the center-off mass ($\underline{r}_0$) and the relative location vector ($\underline{r}_r$):

$$\vec{r}_0 = \frac{1}{2}(\vec{r}_1+\vec{r}_2) \qquad \vec{r}_r = \vec{r}_2 - \vec{r}_1$$



## The distribution of the relative velocity (2)

- The individual velocities:

$$\vec{v}_1 = \dot{\vec{r}}_0 - \frac{1}{2}\vec{v}_r \qquad \vec{v}_2 = \dot{\vec{r}}_0 + \frac{1}{2}\vec{v}_r$$

$$m(v_1^2+v_2^2) = 2m\dot{\vec{r}}_0^{\,2} + \frac{m}{2}v_r^2$$

$$\left|\underline{\underline{J}}\right| = 1$$

$$f_{12}(\vec{v}_r,\vec{v}_0)d\vec{v}_r d\vec{v}_0 = \left(\frac{m}{4\pi kT}\right)^{3/2} e^{-\frac{mv_r^2}{4kT}}dv_{rx}dv_{ry}dv_{rz}\left(\frac{m}{\pi kT}\right)^{3/2} e^{-\frac{mv_0^2}{kT}}dv_{0x}dv_{0y}dv_{0z}$$

## The distribution of the relative velocity (3)

- A product of two factors, depending on relative and center-of-mass, respectively; the first is more important to us:

$$f_r(\vec{v}_r)d\vec{v}_r = \left(\frac{m}{4\pi kT}\right)^{3/2} e^{-\frac{m v_r^2}{4kT}} dv_{rx}dv_{ry}dv_{rz}$$

- And the expected value:

$$\langle v_r \rangle = 4\pi\left(\frac{m}{4\pi kT}\right)^{3/2} \int_0^\infty v_r^3 e^{-\frac{m v_r^2}{4kT}} dv_r = \sqrt{2}\langle v \rangle$$



## Collision number
## Mean free path

$$Z = \int_0^\infty \sigma v_r \frac{N}{V} f(v_r)dv_r === \frac{N}{V}\sigma\langle v_r \rangle = n\sigma\sqrt{2}\langle v \rangle = \frac{4N}{V}\sigma\sqrt{\frac{kT}{\pi m}} =$$
$$= \sqrt{\frac{16\sigma^2}{kT\pi m}}p$$

- The free path is the ratio of the mean velocity and the collision number (a particle travels <v> and collides Z times at a during a time unit):

$$\lambda = \frac{\langle v \rangle}{Z} = \frac{V}{\sqrt{2}N\sigma} = \frac{kT}{\sqrt{2}p\sigma} \qquad \left(\frac{V}{N} = \frac{kT}{p}\right)$$

- The time passed between molecular collisions: $\tau = \frac{1}{Z}$



## The distribution of free path

p(x) is the probability that a particle travels a distance x without collision

$$p(x+dx) = p(x)p(dx) = p(x) + \frac{dp}{dx}dx$$

The probability $\tilde{p}(dx)$ that the particle collides at path dx is proportional to dx

$$\tilde{p}(dx) = \alpha dx$$

$$p(dx) = 1 - \tilde{p}(dx) = 1 - \alpha dx$$

$$p(x+dx) = p(x) + \frac{dp}{dx}dx = p(x)(1 - \alpha dx)$$



## The distribution of free path (2)

$$\frac{dp}{dx} = -\alpha p(x) \longrightarrow p(x) = Ae^{-\alpha x} \quad (\text{limits} \Rightarrow \text{A=1})$$

dP is the probability that the particle travels freely path x then collides on distance dx:

$$dP = p(x)\tilde{p}(dx) = p(x)\alpha dx = \alpha e^{-\alpha x}dx$$

$$\lambda = \int_0^1 x dP = \int_0^\infty x\alpha e^{-\alpha x}dx = \frac{1}{\alpha}$$



## The distribution of free path (3)

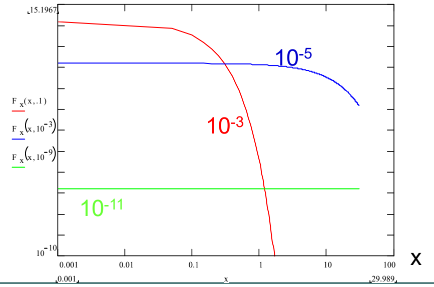
$$dP = \frac{1}{\lambda}e^{-\frac{1}{\lambda}x}dx \longrightarrow F(x) = \frac{1}{\lambda}e^{-\frac{1}{\lambda}x}$$

The time passed between collisions can be calculated analogously, only the integral is different:

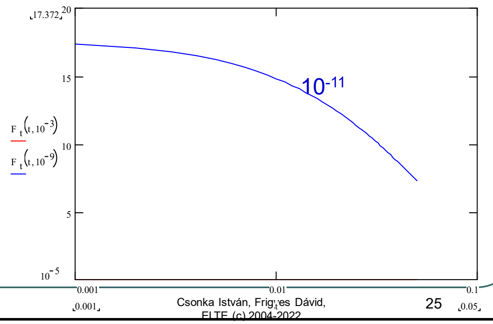
$$F_t(t) = Ze^{-Zt}$$



## Distribution of the free path (4)



## Distribution of the „free time”



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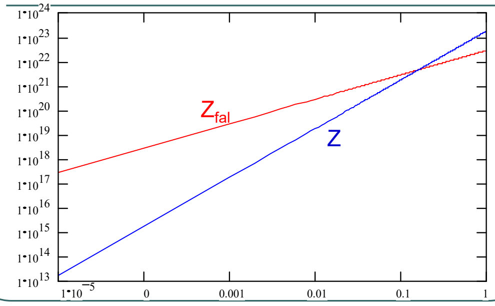
## Molecular and wall-collisions



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## Transport phenomena

- Close to equilibrium but a physical quantity is not homogeneous
- p (momentum): viscosity
- c: diffusion
- T: thermal conductance
- Can be described by means of collision numbers



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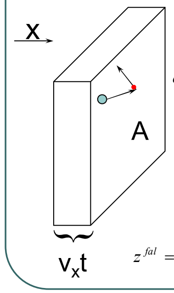
## Number of wall-collisions




$\frac{dN_{v_x}}{V} \cdot A \cdot v_x t$ Molecules reach the wall

$$dN_{v_x} = N \cdot f_x(v_x) dv_x \Longrightarrow z_{v_x}^{fal} = \frac{N}{V} \cdot f_x(v_x) \cdot v_x dv_x$$

$$\int_0^\infty f_x v_x dv_x = \sqrt{\frac{m}{2\pi \cdot kT}} \int_0^\infty v_x e^{-mv_x^2/2kT} dv_x =$$

$$= \sqrt{\frac{m}{2\pi \cdot kT}} \frac{kT}{m} = \sqrt{\frac{kT}{2\pi \cdot m}} = \frac{1}{4}\langle v\rangle$$

$$z^{fal} = \frac{N}{4V} \cdot \langle v\rangle = \frac{N}{V}\sqrt{\frac{kT}{2\pi \cdot m}} = \frac{p}{kT}\sqrt{\frac{kT}{2\pi \cdot m}} = \frac{p}{\sqrt{2\pi \cdot mkT}}$$



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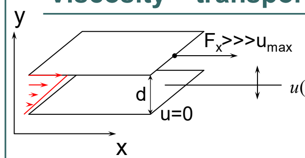
## Viscosity – transport of momentum




$$u(y) = \frac{u_{max}}{d} y \qquad \eta = \frac{F_x}{A \cdot \frac{u_{max}}{d}}$$

Number of particles reaching the wall (/t/A): $z^{fal} = \frac{\langle v\rangle}{4}\frac{N}{V}$

Their momentum in direction x: $\frac{\langle v\rangle}{4}\frac{N}{V} m u_{max} \frac{\lambda_0}{d} \stackrel{?}{=} \frac{\langle v\rangle}{3}\rho u_{max}\frac{\lambda}{d}$

The force: $\frac{F_x}{A} = \frac{\langle v\rangle}{3}\rho u_{max}\frac{\lambda}{d} \Longrightarrow \eta = \frac{\langle v\rangle}{3}\rho\lambda \sim \sqrt{T}$



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## Discussion

- This formula holds except for
  - High pressure (starting point of kinetic gas theory is not true), but this is not the case in vacuum
  - Low pressure: when λ>d, the actual value of λ does not matter



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## Viscosity at pure molecular flow

- The particle travels without collisions, thus there is no linear velocity distribution in direction y but the particle obtains the momentum of the surfaces:

$$\frac{F_x}{A} = \frac{1}{\beta}\frac{\langle v\rangle}{4}\rho u_{\max}$$

Close to 1



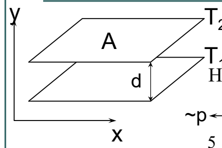
## Transition (Knudsen) range

- The two effects (free travel between walls and linear velocity distribution) apply together

$$\frac{F_x}{A} = \frac{1}{\beta + \alpha\frac{d}{\lambda}}\langle v\rangle\rho\cdot u_{\max} \sim \frac{p}{1+\frac{\alpha}{\beta}p}$$




## Thermal conductivity
## The transport of kinetic energy




$$T(y) = T_1\frac{\Delta T}{d}y$$

Heat flux/hőáramsűrűség

power

$$j_q = \frac{P}{A} = -\kappa\frac{\Delta T}{d}$$

~p

$\kappa = \frac{5}{6}k\lambda\langle v\rangle\frac{N}{V}$: independent of pressure (limits)

~1/p

$$\langle v_x^2\rangle = \frac{kT}{m}$$

$$\langle E_{kin}\rangle = \frac{3}{2}NkT \longrightarrow c_V = \frac{C}{V} = \frac{E}{TV} = \frac{3}{2}\frac{Nk}{V}$$

$\kappa = \frac{5}{9}\lambda\langle v\rangle c_V$ κ is proportional to η; $c_v$ breaks the law at higher range



## Free molecular thermal conductivity

The particle obtains part of the energy of the heated surface.
Accommodation coefficient:

$$\alpha = \frac{T_r - T_1}{T_2 - T_1}$$

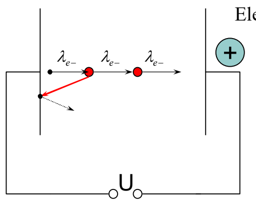
| α | $H_2$ | $N_2$ |
|---|---|---|
| plain Pt | 0,36 | 0,89 |
| Pt black | 0,71 | 0,95 |
| W | 0,20 | 0,57 |

$$j_q = \frac{P}{A} = \frac{\langle v\rangle_1}{4}\frac{N}{V}2k(T_r - T_1) = \frac{\langle v\rangle_1}{2}\frac{p}{T_1}(T_r - T_1) = \frac{\alpha\langle v\rangle_1}{2}\frac{p}{T_1}(T_2 - T_1)$$



## Electrical conductivity

$$\lambda_{ion} = \sqrt{2}\lambda \qquad \lambda_{e-} = 4\lambda \qquad \langle KE_e\rangle = e\frac{\lambda_{e-}}{d}U$$



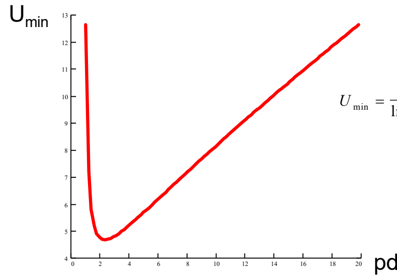

Electron avalanche: $IE \le e\frac{\lambda_{e-}}{d}U$

$$U_{\min} = \frac{IE\cdot d}{\lambda\cdot e} \sim pd$$

Continuous discharge: avalanche + electron supply
Low pressure: lack of charge carrier



## Paschen-curve




$$U_{\min} = \frac{Apd}{\ln(pd) + B}$$

## Types of electric discharge

- Spark (short time, e.g., conductors moving away)
- Arc (continuous electron supply)
- Glow discharge (low pressure, glow)
- Corona discharge (partial discharge around sharp conductor)



## Gas flow – particle transport

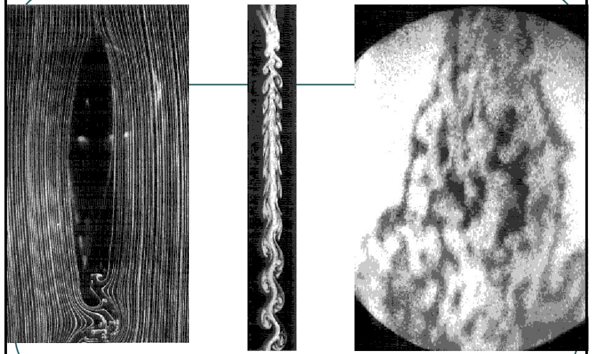
- Viscous flow: Reynolds-number:
  - Laminar flow (Re<1200)
  - Turbulent flow (Re>2200)

$$\mathrm{Re} = \frac{du\rho}{\eta}$$

- Knudsen-number:
  - Molecular flow (Kn>1)
  - Transition regime (1>Kn>0,01)
  - Viscous flow (Kn<0,01)

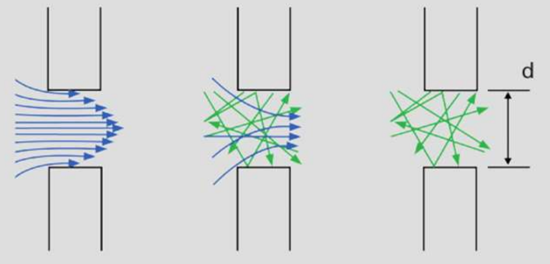
$$Kn = \frac{\lambda}{d}$$





laminar transition turbulent






**Continuous flow**
Kn < 0.01
Low vacuum

**Knudsen flow**
0.01 < Kn < 0.5
Medium vacuum

**Molecular flow**
Kn > 0.5
High/Ultra-high vacuum

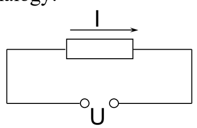


| | Viscous flow | Knudsen flow | Molecular flow |
|---|---|---|---|
| | Low vacuum | Medium vacuum | High // Ultra-high vacuum |
| Pressure range [hPa] | $10^3 \ldots 1$ | $1 \ldots 10^{-3}$ | $< 10^{-3}$ // $< 10^{-7}$ |
| Pressure range [Pa] | $10^5 \ldots 10^2$ | $10^2 \ldots 10^{-1}$ | $< 10^{-1}$ // $< 10^{-5}$ |
| Knudsen number | $Kn < 0.01$ | $0.01 < Kn < 0.5$ | $Kn > 0.5$ |
| Reynolds number | Re < 2,300: laminar<br>Re > 4,000: turbulent | | |
| p · d [hPa cm] | p · d > 0.6 | 0.6 > p · d > 0.01 | p · d < 0.01 |



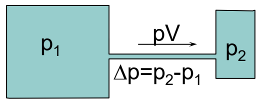
## Conductance

Analogy:




$$R = \frac{U}{I} \qquad G = \frac{1}{R} = \frac{I}{U}$$

$$I = \frac{dQ}{dt} = G\Delta U = \frac{\Delta U}{R}$$




$$\frac{d}{dt}pV = \frac{\Delta p}{Z} = C\Delta p$$

$$[C] = \frac{[V]}{[t]} = \frac{l}{s}$$

## Resultant conductance

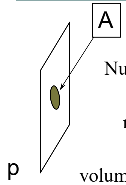
Series: $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$

Parallel: $C = C_1 + C_2 + \cdots$

Conductance depends on:
- Type of flow (molecular, Knudsen, laminar, turbulent)
- pressure (independent in molecular flow)
- M
- T



## Flow through a hole (molecular!!!)




Number of particles / time unit: $z^{fal} = A\frac{N}{4V}\cdot\langle v\rangle$

mass: $mz^{fal} = A\frac{N}{4V}m\cdot\langle v\rangle = A\frac{\langle v\rangle}{4}\rho$

volume: $A\frac{\langle v\rangle}{4}$ Gas amount: $Q = pV = pA\frac{\langle v\rangle}{4}$

Flowing into non-zero p (backflow):

$$Q = A\frac{\langle v\rangle}{4}\Delta p = \frac{\langle v\rangle}{4}\frac{d^2\pi}{4}\Delta p = A\frac{1}{4}\sqrt{\frac{8kT}{\pi m}}\Delta p$$



## Examples

- Long tube, laminar flow, Poiseuille-law:

$$Q = \overbrace{\frac{\pi\cdot d^4}{128\eta\cdot L}p_0}^{C}(p_1 - p_2)$$

- Long tube, molecular flow:

$$C = \frac{16}{3}\sqrt{\frac{kT}{2m\pi}}\frac{A^2}{o}L$$

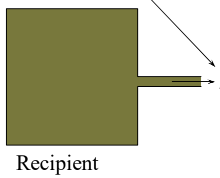


## Flow through hole (23°C, air)

- Viscous flow ($p_1/p_2$=0,5): $C_{viszk} = 40A\frac{l}{s}$
- Viscous flow ($p_1/p_2$=1): $C_{viszk} = 20A\frac{l}{s}$
- Molecular flow: $C_{mol} = 11{,}6A\frac{l}{s}$



## Pumping speed, Throughput



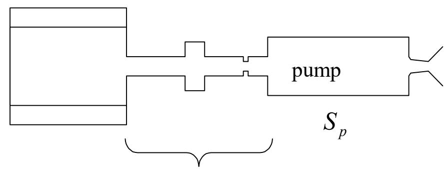

$S_p = \frac{dV}{dt}$

$Q = pS_p = p\frac{dV}{dt} = C\Delta p$

Throughput is identical in the whole instrument (energy dimension)



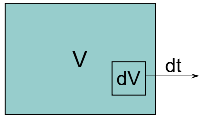
## Effective pumping speed




$\frac{1}{C_{tot}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$

$\frac{1}{S_{eff}} = \frac{1}{S_p} + \frac{1}{C_{tot}}$ Important in design/implementation!

## Pumping speed




$$S = \frac{dV}{dt}$$

$$dp = -\frac{dV}{V} \cdot p = -\frac{Sdt}{V} \cdot p$$

$$p = p_0 e^{-\frac{S}{V}t} \xrightarrow{(t=\frac{V}{S})} p = \frac{p_0}{e}$$

Pumps
- comprimation
- trapping

True only when volume pumping (outgassing!!!)



## Basic equation of vacuum technology

$$V\frac{dp}{dt} = I_{be} - pS_p$$

$I_{be}$
- holes
- desorption
- evaporation

gas load
Statical/dynamic vacuum system

Final vacuum:
Pressure is no longer decreasing:

$$V\frac{dp}{dt} = I_{be} - pS_p = 0$$

$I_{be} = pS_p$ ill.: $I_{be} = pS_{eff}$